



GCE AS MARKING SCHEME

SUMMER 2025

**AS
FURTHER MATHEMATICS
UNIT 3 FURTHER MECHANICS A
2305U30-1**

About this marking scheme

The purpose of this marking scheme is to provide teachers, learners, and other interested parties, with an understanding of the assessment criteria used to assess this specific assessment.

This marking scheme reflects the criteria by which this assessment was marked in a live series and was finalised following detailed discussion at an examiners' conference. A team of qualified examiners were trained specifically in the application of this marking scheme. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners. It may not be possible, or appropriate, to capture every variation that a candidate may present in their responses within this marking scheme. However, during the training conference, examiners were guided in using their professional judgement to credit alternative valid responses as instructed by the document, and through reviewing exemplar responses.

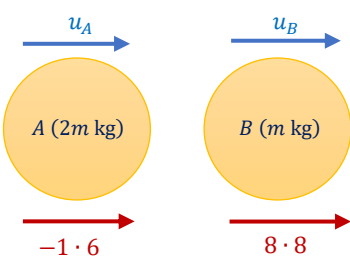
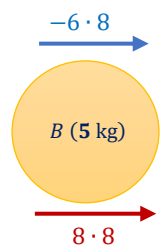
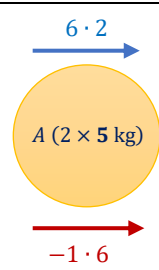
Without the benefit of participation in the examiners' conference, teachers, learners and other users, may have different views on certain matters of detail or interpretation. Therefore, it is strongly recommended that this marking scheme is used alongside other guidance, such as published exemplar materials or Guidance for Teaching. This marking scheme is final and will not be changed, unless in the event that a clear error is identified, as it reflects the criteria used to assess candidate responses during the live series.

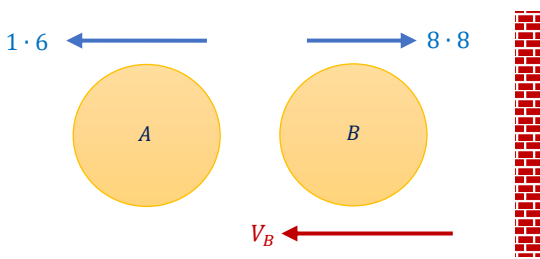
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UNIT 3 FURTHER MECHANICS A

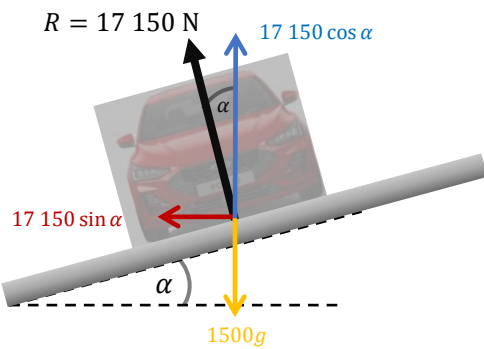
SUMMER 2025 MARK SCHEME

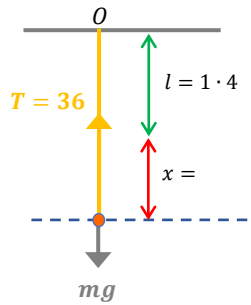
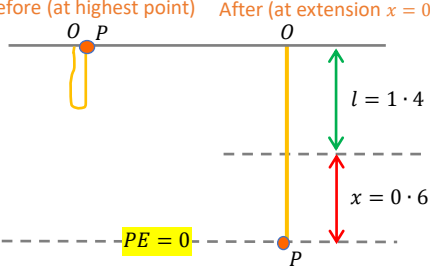
Q1	Solution	Mark	Notes
(a)	$\mathbf{a} = \frac{d\mathbf{v}}{dt}$ $\mathbf{a} = 6t^2\mathbf{i} + 4\mathbf{j} - 2t\mathbf{k}$ $\mathbf{F} = m\mathbf{a} = 1 \cdot 5(6t^2\mathbf{i} + 4\mathbf{j} - 2t\mathbf{k})$ $\mathbf{F} = 9t^2\mathbf{i} + 6\mathbf{j} - 3t\mathbf{k}$	M1 A1 B1 [3]	Correct differentiation of at least one term All correct FT their $\mathbf{a}$, isw
(b)	$\mathbf{F} \cdot \mathbf{v} = (9t^2 \times 2t^3) + (6 \times 4t) + (-3t \times -t^2)$ $\mathbf{F} \cdot \mathbf{v} = 18t^5 + 24t + 3t^3$ At $t = 2$, $\mathbf{F} \cdot \mathbf{v} = 18(2)^5 + 24(2) + 3(2)^3$ $\mathbf{F} \cdot \mathbf{v} = 648 \text{ W}$	M1 m1 A1 [3]	Correct method for dot product Must lose $\mathbf{i}, \mathbf{j}, \mathbf{k}$ FT F from part (a) Use of $t = 2$ cao, including units (oe) For example, J s^{-1} $\text{kg m}^2\text{s}^{-3}$ N m s^{-1}
	<u>Alternative Solution</u> At $t = 2$, $\mathbf{v} = 16\mathbf{i} + 8\mathbf{j} - 4\mathbf{k}$, $\mathbf{F} = 36\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$ $\mathbf{F} \cdot \mathbf{v} = (36 \times 16) + (6 \times 8) + (-6 \times -4)$ $\mathbf{F} \cdot \mathbf{v} = 648 \text{ W}$	(M1) (m1) (A1) ([3])	Use of $t = 2$ for both FT F from part (a) Correct method for dot product Must lose $\mathbf{i}, \mathbf{j}, \mathbf{k}$ cao, including units (oe)
Total for Question 1		6	

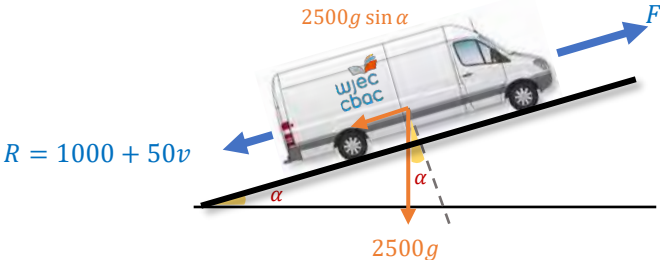
Q2	Solution	Mark	Notes
(a)			$e = 0.8$
	Conservation of momentum $(2m)(u_A) + (m)(u_B) = (2m)(-1.6) + (m)(8.8)$ $2u_A + u_B = 5.6$ Restitution $8.8 - (-1.6) = -0.8(u_B - u_A)$ $u_A - u_B = 13$ Elimination of one variable $u_A = 6.2 \Rightarrow \text{speed of A} = 6.2 \text{ (ms}^{-1}\text{)}$ $u_B = -6.8 \Rightarrow \text{speed of B} = 6.8 \text{ (ms}^{-1}\text{)}$	M1 A1 M1 A1 m1 A1 A1 [7]	Attempted. Allow 1 sign error All correct Attempted. Allow 1 sign error All correct, oe (Ratio) cao cao
(b)			
	Impulse on B (Impulse on A) $I_B = m_B v_B - m_B u_B \quad -I_A = m_A v_A - m_A u_A$ $= (5)(8.8 - -6.8) \quad = (10)(-1.6 - 6.2)$ $= 78 \quad = -78$ $I_B = 78 \text{ (Ns)}$ Opposite direction to original motion of B	M1 A1 B1 [3]	Used, allow 1 sign error cao cao Accept • 'to the right' • $\rightarrow$ • Towards the wall

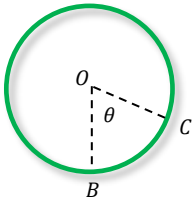
Q2	Solution	Mark	Notes
(c)			
	$V_B = \pm 8 \cdot 8e$ To collide with A again, $V_B > 1 \cdot 6$ $8 \cdot 8e > 1 \cdot 6$ $e > \frac{2}{11} \quad (= 0 \cdot 1818 \dots)$ Range: $\frac{2}{11} < e \leq 1$	B1 M1 A1 [3]	si ANY comparison of $\pm 8 \cdot 8e$ with $\pm 1 \cdot 6$ cao
Total for Question 2		13	

Q3	Solution	Mark	Notes
(a)	Position vector of the form $\mathbf{r} = \mathbf{p} + \mathbf{v}t$ $\mathbf{r}_A = \begin{cases} 3\mathbf{i} - 10\mathbf{j} + 20\mathbf{k} + (-\mathbf{i} + 2\mathbf{j} + \mathbf{k})t \\ \begin{pmatrix} 3-t \\ -10+2t \\ 20+t \end{pmatrix} \\ (3-t)\mathbf{i} + (-10+2t)\mathbf{j} + (20+t)\mathbf{k} \end{cases}$ $\mathbf{r}_B = \begin{cases} -2\mathbf{i} + 30\mathbf{j} + 15\mathbf{k} + (3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})t \\ \begin{pmatrix} -2+3t \\ 30-4t \\ 15+2t \end{pmatrix} \\ (-2+3t)\mathbf{i} + (30-4t)\mathbf{j} + (15+2t)\mathbf{k} \end{cases}$	M1 A1 A1 [3]	oe isw when correct result seen isw when correct result seen
(b)	$AB = \mathbf{r}_B - \mathbf{r}_A$ $= (-5 + 4t)\mathbf{i} + (40 - 6t)\mathbf{j} + (-5 + t)\mathbf{k}$ $AB^2 = (-5 + 4t)^2 + (40 - 6t)^2 + (-5 + t)^2$ $AB^2 = 53t^2 - 530t + 1650$	M1 A1 m1 A1 [4]	$\pm(\mathbf{r}_B - \mathbf{r}_A)$ Attempted difference for all 3 terms oe, at least 2 correct terms oe. Must lose i, j, k and be linear expressions Convincing, intermediate line required
(c)	$\frac{d}{dt} AB^2 = \frac{d}{dt} (53t^2 - 530t + 1650) (= 106t - 530)$ $\frac{d}{dt} AB^2 = 0 \quad (106t - 530 = 0)$ $t = 5$ When $t = 5$, $AB^2 = 53(5)^2 - 530(5) + 1650$ $(= 325)$ Least distance is $\sqrt{325} = 5\sqrt{13} = 18.02(775638) \quad (\text{m})$	M1 m1 A1 A1 [4]	First derivative attempted No FT as quadratic given in (b) Used cao cao, any form
	<u>Alternative solution</u> $AB^2 = 53(t^2 - 10t) + 1650$ $AB^2 = 53(t - 5)^2 + 325$ Minimum value occurs when $t = 5$ When $t = 5$, $AB^2 = 325$ Least distance is $\sqrt{325} = 5\sqrt{13} = 18.02(775638) \quad (\text{m})$	(M1) (m1) (A1) (A1) [(4)]	Attempt to 'complete the square' $53(t \pm a)^2 + b$, allow $b = 0$ cao cao, any form
Total for Question 3		11	

Q4	Solution	Mark	Notes
(a)	 Resolve vertically $R \cos \alpha = mg$ $17\,150 \cos \alpha = 1500g$ $\cos \alpha = \frac{6}{7} = \frac{14700}{17150}$ $\alpha = 31.00271913 \dots (\approx 31^\circ)$	M1 A1 A1 [3]	$\cos \alpha = \frac{6}{7}$ $\sin \theta = \frac{\sqrt{13}}{7}$ $1500g = 14\,700$ Dimensionally correct eqn. Accept $R \sin \alpha = mg$ Convincing
(b)	N2L towards centre $R \sin \alpha = ma$ $17\,150 \sin \alpha = 1500a$ $17\,150 \sin \alpha = 1500 \left(\frac{v^2}{r} \right)$ $17\,150 \left(\frac{\sqrt{13}}{7} \right) = 1500 \left(\frac{v^2}{150} \right)$ $v^2 = 883.3600625$ $v = 29.72(137383 \dots) \text{ (ms}^{-1}\text{)}$ SC1 Award SC1 for $v = 29.72(137383 \dots)$ via $v^2 = rg \tan \alpha$	M1 A1 m1 A1 [4]	Dimensionally correct eqn. Accept $R \cos \alpha = ma$ Used with $a = \frac{v^2}{r}$ $v^2 = \frac{1715\sqrt{13}}{7} = 245\sqrt{13}$ Note. Using $\alpha = 31^\circ$ $v^2 = 883.2902985$ $v = 29.72(020018 \dots) \text{ (ms}^{-1}\text{)}$
Total for Question 4		7	

Q5	Solution	Mark	Notes
(a)	Use of Hooke's Law $36 = \frac{156 \cdot 8x}{1 \cdot 4}$ $x = \frac{9}{28} \quad \text{or} \quad 0 \cdot 32(142857 \dots) \quad (\text{m})$	M1 A1 [2]	
(b)	Before (at highest point) After (at extension $x = 0 \cdot 6$)  Using expression for $EE = \frac{\lambda x^2}{2l}$ $EE = \frac{156 \cdot 8(0 \cdot 6)^2}{2(1 \cdot 4)} = \frac{504}{25} = 20 \cdot 16$ Using expression for $PE = mgh$ $PE = mg(\pm 2) = \frac{98}{5}m = 19 \cdot 6m = 2mg$ KE = $15 \cdot 12$ (provided in question) Conservation of Energy (before = after) $mg(2) = 15 \cdot 12 + \frac{156 \cdot 8(0 \cdot 6)^2}{2(1 \cdot 4)}$ $19 \cdot 6m = 15 \cdot 12 + 20 \cdot 16$ $19 \cdot 6m = 35 \cdot 28$ $2gm = 35 \cdot 28$ $m = 1 \cdot 8$	M1 A1 M1 A1 M1 A1 [7]	Used A correct expression Used A correct exp. relative to their $PE = 0$. For example, $mg(0 \cdot 32) = 3 \cdot 136m$ $mg(0 \cdot 28) = 2 \cdot 744m$ KE ($15 \cdot 12$), EE and PE all present Any combination of signs All correct, oe Convincing, intermediate line required
Total for Question 5		9	

Q6	Solution	Mark	Notes
(a)			
	$F = \frac{P}{20}$ At constant speed $F = R$ (N2L with $a = 0$) $1000 + 50(20) = \frac{P}{20} (= F) \quad 2000 = \frac{P}{20} (= F)$ $P = 40\,000 \quad (W)$	B1 M1 A1 A1 [4]	Used, oe, si $F - R = ma$ with $a = 0$ $1000 + 50v = F$ Correct equation using $v = 20$ cao, oe
(b)	 $2500g \left(\frac{2}{49} \right) = 1000$		
	$F = \frac{100 \times 1000}{v} = \frac{100000}{v}$ Using N2L up slope $F - R - mg \sin \alpha = ma$ $\frac{100000}{v} - (1000 + 50v) - 2500g \left(\frac{2}{49} \right) = 2500(0.3)$ $\frac{100000}{v} - (1000 + 50v) - 1000 = 750$ Forming and solving a quadratic $100000 - 2750v - 50v^2 = 0$ $v^2 + 55v - 2000 = 0$ $(v - 25)(v + 80) = 0$ $v = 25 \quad (\text{ms}^{-1}) \quad (\text{or } v = -80)$	B1 M1 A1 A1 m1 A1 [6]	si All forces, dim. correct. Allow sign errors and use of $mg \cos \alpha$ Correct equation oe, equation in v . cao $v = -80$ must be clearly discarded (as it leads to $R = -3000$)
Total for Question 6		10	

Q7	Solution	Mark	Notes
			$r = 0.5$ $m = 0.04$
(a)	(i) Conservation of energy $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgr(1 - \cos \theta)$ $v^2 = u^2 - 2g(0.5)(1 - \cos \theta)$ $v^2 = \begin{cases} u^2 - g(1 - \cos \theta) = u^2 - 9.8(1 - \cos \theta) \\ u^2 - g + g \cos \theta = u^2 - 9.8 + 9.8 \cos \theta \end{cases}$ (ii) N2L towards centre $\pm(R - mg \cos \theta) = ma$ $R - mg \cos \theta = \frac{mv^2}{r}$ $R = \frac{0.04}{0.5}(u^2 - 9.8 + 9.8 \cos \theta) + (0.04)(9.8) \cos \theta$ $R = 0.08u^2 - 0.784 + 1.176 \cos \theta$	M1 A1 A1 A1 M1 A1 m1 A1	KE and PE in dim. correct equation $\pm \frac{1}{2}mu^2 = \pm \frac{1}{2}mv^2 \pm mgr(\text{trig})$ KE, both terms PE, $mgr(1 - \cos \theta)$ Allow in terms of g Dim. correct equation $mg \cos \theta$, R opposing. Allow $mg \sin \theta$ Correct equation with $a = \frac{v^2}{r}$ Substitute for v^2 Convincing, intermediate line required
	(iii) Let $R = 0$ and $\theta = 120^\circ$ $u^2 = 17.15 = \frac{343}{20}$ $u = 4.14(12558 \dots) \quad \text{or} \quad = \frac{7\sqrt{35}}{10} \quad (\text{ms}^{-1})$	M1 A1 [10]	Used cao
(b)	Work done overcoming resistance $WD = 0.03 \times 10 \quad (= 0.3)$ Work-energy principle $\frac{1}{2}(0.04)(5)^2 - 0.3 = \frac{1}{2}(0.04)v^2$ $0.5 - 0.3 = 0.02v^2$ $v^2 = 10$ $v = \sqrt{10} = 3.1(6227766 \dots) \quad (\text{ms}^{-1})$ $v = 3.1(62277 \dots) < 4$, so the ball will drop into the hole.	B1 M1 A1 A1 [4]	Used, si, $F \times d = E$ All terms included, oe FT their WD oe, i.e. comparison of squares Convincing
Total for Question 7		14	